Class Teaching and Individual Instruction

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Abstract

How does class size affect the way teachers teach? Possibilities to change teaching practices are likely to be the major mechanism to explain the effects of class size on student achievement. Hence, education is modelled as a production function in which class teaching and individual instruction are inputs. Class size reduction enables the teacher to increase both the time devoted to class teaching and the time spent separately on each pupil, explaining the expected positive impact of smaller classes on achievement. The optimal adjustment of teaching practice depends on the complementarity of both teaching methods. It is shown that if teachers' preferences for both teaching methods differs from the actual production function, the effects of class size reduction can be reduced substantially and even become negative. Based on data from the California Class Size Reduction (CSR) and the international mathematics and science study TIMMS it is shown that teachers strongly substitute class teaching for individual instruction. The analyses suggest that overvaluation of individual instruction may be a cause of the small or negative effects of reduced class size which are typically found in both cross-sectional analyzes and class size experiments.

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1 Introduction

Although class size is often regarded as a key variable to influence the effectiveness of education, research has consistently failed to show a large impact of reduced class size on educational performance. At best, experimental studies[1] and studies based on instrumental variables[2] suggest a small positive average impact of class size reduction on the performance, while many cross-sectional investigations even show a negative relationship.[3] Krueger (1999) has shown that behind the small positive average effect found in the STAR experiment there is a large variation of effects between schools. While many schools gained performance from class size reduction, others lost, suggesting that some teachers are more adept in translation smaller classes into achievement than others. Many authors suggest that an inadequate respons of teachers to the opportunities of smaller classes could explain the lack of substantial effects.

In this paper I elaborate on this issue, by developing and estimating a model of the way the way teachers teach and the effects this has for educational performance. The two main features of the model are that (i) the effectiveness of teaching depends on the way a teacher divides his time over two teaching methods – class teaching and individual instruction – and (ii) teacher's ideas about the effectiveness of different teaching methods might differ from the actual effectiveness. Of course, if the way a teacher divides his time over the two teaching methods differs from the optimum of the production function, pupil achievement will decrease. More important how-

[1]Boozer and Cacciola (2001), Krueger (1999), Krueger and Whitmore (1999).

[2]Hoxby (2000a), Angrist and Lavy (1999), Wößmann and West (2002).

[3]See Hanushek (1986, 1997, 1998) for overviews suggesting that in most cases no or a negative impact of size reduction is found. Krueger (2000) argues that these cross-sectional analyses provide evidence for a negative effect of class size on achievement.

ever, based on a CES production function with class teaching and individual instruction as inputs, it can be shown that if the elasticity of substitution between both teaching methods is larger than 1, class size reduction might substantially increase inefficiency, so pupil achievement benefits less from class size reduction, compared to a situation in which a teacher optimally divides his time over both teaching methods. If a teacher undervalues class teaching, class size reduction might even decrease pupil achievement.

The intuition behind the model is that a class size reduction by a certain percentage saves the teacher the same percentage of the time he used for individual instruction (income effect). He can use this additional time for extra class teaching and/or for extra individual instruction. When the elasticity of substitution of both teaching methods is larger than 1, the teacher will increase the relative proportion of individual teaching (substitution effect) and perhaps even decrease the time spent on class teaching to increase the time for individual instruction more than is allowed for by the time saved by size reduction only. If a teacher bases his teaching style on incorrect valuations of the effectiveness of the two teaching methods, the relative gain of class size reduction can be reduced. This perverse effect is especially large when substitutability is large and teachers ascribe too much value to individual instruction. It can be shown that the accumulation of effects might cause a decrease in achievement due to reduced class size.

Based on data from the California Class Size Reduction and TIMMS it is shown that teachers indeed strongly substitute class teaching for individual instruction. As a result pupils loose substantially more hours of class teaching than they gain in individual instruction. Furthermore, it is shown that the division between class teaching and individual instruction depends strongly on the personal opinions of the teachers about education. The analyses show that – especially for teachers who prefer individual instruction – the overvaluation of individual teaching, could be the cause of a small or negative effect of class size reduction.

These findings lead to the question why teachers would overvalue individual instruction. In section 5 I discuss several possible reasons. Besides a sheer lack of knowledge of teachers of effective teaching, teachers might derive more work satisfaction from individual instruction, there might be hidden educational targets, achievement of low-achievers might be valued higher than the achievement of others and due to the external effect of asking individual attention it might be costly to reach the optimal amount of class teaching.

The model enables a comparison between class size effects and time-on-task. The time gain of a class size reduction equals the time saved by the reduction of the number of pupils who get individual instruction. The way in which this time saving is reallocated determines the gain in achievement. Furthermore, the model shows that if teachers value both teaching methods adequately, class size reduction will regain its expected substantial positive effect in classes where individual instruction is important. This means that there is not a choice between teacher training or improved teacher incentives and class size reduction, but rather that for class size reduction to become effective, first an adequate teaching style has to established.

This paper is related to empirical studies about the effectiveness of reducing class size. The poor results found in studies on class size reduction has led some researchers to argue that other policies to increase educational performance, such as teacher training (Angrist and Lavy 1999, 2001), computer facilities (Betts 1995), increased incentives for teachers (Lavy 1999a) and extended length of the hours children receive lessons (time-on-task, Lavy (1999c)) might be more effective. According to Rivkin et al. (1998) the benefits of increased quality of teachers are much larger than possible benefits of class size reduction. Therefore, if the large increase in demand for teachers due to class size reduction leads to the recruitment of less able teachers, the disadvantages might outweigh the benefits. The observation that some alternatives provide better results per invested dollar, however, does not explain why the strategy of class size reduction which is appealing to many teachers and parents does not lead to the expected effects.

Related to this, others have argued that one can not expect any effect of class size reduction if teachers don't change the way they teach. The success of a class size reduction depends on the way teachers react to the new circumstances. As Stasz and Stecher (1999) put it:

> "Many believe that the most important benefit of reducing class size is the opportunity it provides for teachers to get to know their students better and to work with students one-to-one. Interview data from our case studies suggest that smaller classes did have this effect: teachers in reduced size classes uniformly said that having fewer students enabled them to get to know each one better and sooner. However, ..., survey data indicate that teachers in non-reduced and reduced size classes spent similar amounts of time, on average, working with individual students in the course of their language arts and mathematics instruction." (p. 97)

Cahen et al. (1983) investigated a couple of classes experiencing a substantial reduction in class size during the year and concluded that the instruction practices of teacher did not really change. In a similar study in Canada, Shapson et al. (1980) concluded that teachers did not change the proportion of time allocated to the whole class, or individual practices. Pong and Aaron (2001) compare results from the TIMMS 94-95 survey of 9 countries and conclude that (except for Hong Kong) there is little evidence "that smaller or larger classes differ in ... the instructional practice of teacher." In contrast with these studies, Blatchford and Mortimore (1994), Rice (1999) and Smith and Glass (1980) find that in smaller classes more attention is given to individual students. Stasz and Stecher (1999) add to this that especially low-achieving students benefit most in individual attention of the teacher when class size is reduced. This paper adds to these studies the finding that even if a similar amount of time is spend on individual instruction, reducing class size increases the amount of time spend on individual instruction per student. Therefore, a modest increase in total time spend on individual instruction implies a very large increase in individual instruction per student accompanied by a decrease in class teaching.

My analyis is also related to Lazear (2001), who analyzed theoretically the effects of class size by explicitly introducing a production function. Loss in achievement due to larger classes is explained in this production function by the probability of each student to disrupt the whole class. It is shown that when schools maximizes achievement per dollar spent, schools with better (i.e. less disruptive) students have larger classes and higher levels of achievement. In this way, Lazear is able to explain the cross-sectional positive correlation between class size and achievement. In equilibrium the effect of a marginal reduction in class size on achievement might be small, explaining to modest effects of class size reduction typically found in experimental studies or analyses based in instrumental variables. Although Lazear allows for a wide interpretation of disruptive behavior, including individual questions, he assumes that these moments have no educational value. In this paper, I take into account that effective teaching requires both class teaching and individual instruction. In addition, by allowing teachers having sub-optimal teaching practices it is shown that not only differences between school but also changes in the size of a given class might explain the positive correlation between class size and achievement.

The remainder of this paper is organized as follows. In Section 2 the production function model is introduced and the implications of a discrepancy between teachers' educational preferences and the actual production function are analyzed. In Section 3 the data are described. Section 4 provides the empirical results. In Section 5 I discuss why teachers' behavior might differ from what is optimal from the production function point of view. Section 6 concludes.

2 Class Size and Teaching Style

In this section a model to analyse the effects of class size on teaching behavior will be developed. First, a production function of pupil achievement is introduced and the teacher's optimal behavior is derived. Next, the model will be extended by including the possibility that teachers value the two methods of teaching different from their actual impact in the production function. In this framework the effects of class size reduction on pupil achievement are investigated.

Basic Model

Assume that in a class a teacher can divide his time over *class teaching* and *individual instruction*. All pupils jointly benefit from class teaching, while the teacher can provide individual instruction only to one pupil at a time. The amount of time devoted to class teaching is indicated by c, and the amount of time each pupil receives individual instruction by a. Assuming that every pupil in a class receives an equal amount of individual instruction, the teacher has to divide his time over c class teaching and na individual instruction, in which n denotes class size. Pupil achievement (Y) is related to the way in which the lessons are organized according a CES production function

$$Y = \left(\gamma_a a^\rho + \gamma_c c^\rho\right)^{1/\rho}, \tag{1}$$

in which the elasticity of substitution $\sigma = \frac{1}{1-\rho}$.

By normalizing the time available for teaching to 1, it follows that $c = 1 - an$, so achievement equals

$$Y = \left(\gamma_a a^\rho + \gamma_c (1 - na)^\rho\right)^{1/\rho}. \tag{2}$$

Given the value of each unit of achievement (τ) and the wage costs of a teacher (w), the optimal class size can be found by maximizing $\tau Y - \frac{w}{n}$ with respect to a and n.[4] The optimal n depends on the parameters of the production function, the wage of a teacher and the monetary valuation of pupil achievement. In this model the focus will not be on this optimal class

[4]Lazear (2001) shows in a model based on class room disruptions that a class with better behaved students has a larger optimal class size combined with a better educational performance, explaining a positive correlation between class size and performance in cross-section data. In this model the assumption is made that both teaching methods have a positive value. As a consequence, once it is optimal to restrict teaching almost completely to class teaching, very large optimal class sizes might result. Lazear's disruption arguments provide a very plausible argument why very large classes are not common in primary and secondary education. Since in this paper n is treated exogenously, such arguments do not qualitatively change the results. Therefor, total teaching time has to be understood as total effective teaching time, i.e. without the moments of disruption.

size, but the effects of exogenous shifts in the class size are investigated. For the optimal amount of individual instruction, given class size, the derivative of pupil achievement with respect to the time spend on individual instruction has to equal 0, so

$$\frac{\partial Y}{\partial a} = (\gamma_a a^\rho + \gamma_c(1-na)^\rho)^{\frac{1-\rho}{\rho}} \left(\gamma_a a^{\rho-1} - n\gamma_c(1-an)^{\rho-1}\right) = 0. \quad (3)$$

From this it follows that, given class size, teachers' wages and the valuation of achievement do not matter for the optimal division of time. Furthermore, $\theta = a/(1-an)$, i.e. the odds of the time each child receives individual instruction and class teaching, equal

$$\theta = \frac{a}{1-an} = \left(\frac{n\gamma_c}{\gamma_a}\right)^{-\sigma} = (n\gamma)^{-\sigma}, \quad (4)$$

with $\gamma = \gamma_c/\gamma_a$.

This leads to the following expression for the optimal time each child receives individual instruction, the time a teacher gives individual instruction and the time a teacher gives and thus a child receives class teaching:

$$a = \frac{n^{-1}}{1+n^{\sigma-1}\gamma^\sigma}, \quad (5)$$

$$an = \frac{1}{1+n^{\sigma-1}\gamma^\sigma}, \quad (6)$$

$$c = 1-an = \frac{n^{\sigma-1}\gamma^\sigma}{1+n^{\sigma-1}\gamma^\sigma}. \quad (7)$$

If $\sigma > 1$,(c) is increasing in n, and thus will class size reduction lower the amount of class teaching. Substituting the optimal division of time in the production function leads to:

$$Y^* = \frac{\gamma_a^{\frac{\sigma}{\sigma-1}}}{n} \left(1 + \gamma^\sigma n^{\sigma-1}\right)^{\frac{1}{\sigma-1}}, \tag{8}$$

and the derivative of achievement with respect to class size then equals

$$\frac{\partial Y^*}{\partial n} = \frac{Y^*}{n} \frac{-1}{1 + n^{\sigma-1}\gamma^\sigma} = -Y^* a \tag{9}$$

This derivative is always negative, reflecting that if a teacher responds optimally to class size, class size reductions will always lead to improved achievement. A change in the class size from n_1 to n_2 saves $(n_1 - n_2)a$ time, which can be used to increase the amount of class teaching, or the amount of individual instruction per pupil. The way in which these additional resources are used depends of course on the parameters of the production function. If a teacher divides his time optimally, both teaching methods have an equal marginal value, so the added value of a marginal reduction in class size does not depend on the way the extra time is used. This feature of the model enables a direct comparison between class size reduction and an extension of the hours on task. With an equivalent increase in resources to increase school hours per week $\frac{n_1 - n_2}{n_1}$ time is gained, so the ratio between the gain from class size reduction and extended school hours equals an, i.e. the fraction of time used for individual instruction. If achievement is linear in school hours, extension of school hours is always a better option than class size reduction. Only with a decreasing return to school hours – due to decreasing concentration and attention of the pupils – class size reduction might become more profitable. This would be the case if the achievement elasticity of school hours would be less than an.

To compare the effect of class size reduction at different parameter values, consider the achievement elasticity of of class size: which equals $-a/n$. An increase in γ will diminish the effect of class size reduction on achievement.

This is due to the fact that if class teaching becomes more important the time gain of smaller classes – which is only achieved for time spent on individual instruction – is smaller. If $\sigma < 1$ the relative effect of class size reduction decreases when n decreases. Thus, when class teaching and individual instruction are highly substitutable, class size reduction is relatively effective in small classes, while in case both teaching methods are more complementary larger classes will profit most from class size reductions.

Discrepancies Between Teaching Style and Optimal Behavior

To analyse the effects of class size reduction when teachers do not optimally respond to a changing situation, I assume that a teacher bases his behavior on a production function that might deviate from the actual production function. To analyse how teachers' valuation of class teaching and individual instruction affects class size reduction, I assume that this production function deviates from the actual production function in its value of γ:

$$\hat{Y} = (\hat{\gamma}_a a^\rho + \hat{\gamma}_c c^\rho)^{1/\rho}\,, \tag{10}$$

with $\hat{\gamma} = \frac{\hat{\gamma}_c}{\hat{\gamma}_a}$.

The way a teacher divides his time over the two methods of teaching therefore equals,

$$\hat{\theta} = \frac{a}{1-an} = \left(\frac{n\hat{\gamma}_c}{\hat{\gamma}_a}\right)^{\frac{1}{\sigma}} = (n\hat{\gamma})^{\frac{1}{\sigma}}. \tag{11}$$

By substituting the teaching behavior formula in the educational production function (1) leads to:

$$\hat{Y} = \frac{\gamma_a^{\frac{\sigma}{\sigma-1}}}{n} \left(1 + \hat{\gamma}^{\sigma} n^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \left(\frac{1 + n^{\sigma-1}\hat{\gamma}^{\sigma-1}\gamma}{1 + n^{\sigma-1}\hat{\gamma}^{\sigma}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{12}$$

This equation differs in two respects from the formula for achievement for optimal teaching behavior (8). First, the second term contains $\hat{\gamma}$ rather than γ. For this part of the equation an increase in $\hat{\gamma}$ leads to an increase in $\hat{Y}$, simply because a higher value of class teaching, given the value of individual instruction, will increase output. Second, the third term – which equals 1 when $\hat{\gamma} = \gamma$ – corrects output for the suboptimality of the allocation of time. Since $Y*$ equal maximum achievement, pupil achievement at actual behavior will be lower than achievement at optimal teacher behavior ($\hat{Y} < Y^*$).

To look how class size reduction affects achievement when teachers do not divide their time optimally it is important to investigate the derivative of the production function

$$\frac{\partial \hat{Y}}{\partial n} = \frac{\hat{Y}}{n} \left(-\frac{1 + \sigma n^{\sigma-1}\hat{\gamma}^{\sigma}}{1 + n^{\sigma-1}\hat{\gamma}^{\sigma}} + \frac{\sigma n^{\sigma-1}\gamma\hat{\gamma}^{\sigma-1}}{1 + n^{\sigma-1}\gamma\hat{\gamma}^{\sigma-1}}\right). \tag{13}$$

When $\hat{\gamma} = \gamma$ this formula equals (9).

Comparing this derivative with the derivative of achievement at optimal teacher behavior makes it possible to answer the questions:

1. When will the relative gain in achievement caused by class size reduction be lower than could be expected from optimal teacher behavior?

2. Under what circumstances is it possible that class size reduction decreases pupil achievement?

To answer the first question consider the ratio between the relative achievements at actual and optimal behavior:

$$\frac{\frac{\partial \hat{Y}/\partial n}{\hat{Y}/n}}{\frac{\partial Y^*/\partial n}{Y^*/n}} = (1 + \sigma n^{\sigma-1}\hat{\gamma}^{\sigma}) \left(\frac{1 + n^{\sigma-1}\gamma^{\sigma}}{1 + n^{\sigma-1}\hat{\gamma}^{\sigma}} \right) - (\sigma n^{\sigma-1}\gamma\hat{\gamma}^{\sigma-1}) \left(\frac{1 + n^{\sigma-1}\gamma^{\sigma}}{1 + n^{\sigma-1}\gamma\hat{\gamma}^{\sigma-1}} \right) \tag{14}$$

Investigation of this relative effect of class size reduction leads to four different patterns, depending on the values of σ, γ and $\hat{\gamma}$:

$\sigma < 1$ and $\hat{\gamma} > \gamma$ If class teaching and individual instruction are relatively complementary and teachers overestimate the value of class teaching the relative effect of class size reduction is always lower than it would be if the teacher would respond optimally. Since the teacher provides too much class teaching, the time gain of a reduction in class size is smaller and therefore the gain in achievement is smaller too.

$\sigma < 1$ and $\hat{\gamma} < \gamma$ If both teaching methods are complementary, but teachers overestimate the value of individual instruction, the actual relative effect of a class size reduction will always be larger than the effect under optimal circumstances. Although the achievement is always below the optimum, there is a larger time gain from class size reduction due to the large amount of time spend on individual instruction.

$\sigma > 1$ and $\hat{\gamma} > \gamma$ If class teaching and individual instruction are relative substitutes, the effect of class size reduction compared to the optimal response depends on n. If teachers overestimate the value of class teaching, initially a reduction in n will be less effective, since the amount of individual instruction is low. Due to the high degree of substitutability, however, these teachers will increase the time devoted to individual instruction more than the amount they saved due to the class size reduction. Time for class teaching will therefore decrease, so from a certain point on, when classes become sufficiently small, they will catch up and the gap in achievement will decrease.

$\sigma > 1$ and $\hat{\gamma} < \gamma$ If both teaching methods are substitutable, but teachers overvalue individual instruction, they gain much time when class size is reduced, but they will for more than 100 percent spend this time on more individual instruction. Initially, valuable class teaching will be reduced, making the class size reduction less effective. When n decreases, however, the relative value of class teaching becomes lower and the time gain larger. Therefore if n becomes sufficiently small the effect of class size reduction will become larger than under optimal circumstances.

The last case, in which substitutability is high and teachers underestimate the value of class teaching, leads to the most pronounced effects. Since too much time is spend on individual instruction, class size reduction saves much time. In a response, teachers use all this extra time for more individual instruction and due to the substitutability in addition even reallocate part of the time available from class teaching – with a high marginal value – to individual instruction – with a low marginal value.

The second question is whether there van exist a positive relationship between class size and pupil achievement. This will be the case if the derivative of achievement with respect to class size as in (13) will be larger than 0:

$$-\frac{1+\sigma n^{\sigma-1}\hat{\gamma}^{\sigma}}{1+n^{\sigma-1}\hat{\gamma}^{\sigma}}+\frac{\sigma n^{\sigma-1}\gamma\hat{\gamma}^{\sigma-1}}{1+n^{\sigma-1}\gamma\hat{\gamma}^{\sigma-1}} > 0. \tag{15}$$

This function equals 0 when there is a positive $\bar{n}$ such that

$$\bar{n} = \left((\sigma-1)\gamma\hat{\gamma}^{\sigma-1} - \sigma\hat{\gamma}^{\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{16}$$

If such a n exists, the effect of class size reduction on achievement will be negative for all n above $\bar{n}$. From this it follows that such a reversed relationship will occur when:

$$\sigma > 1, \tag{17}$$

and

$$\frac{\hat{\gamma}}{\gamma} < \rho, \tag{18}$$

for values of n which are sufficiently high $(n > \bar{n})$.

INSERT FIGURE 1

Figure 1 provides a numerical illustration of the model. The parameters values have been chosen to give a pronounced picture of the possible effects of class size reduction. The tick line shows pupil achievement as a function of class size, assuming that teachers optimally divide their time over the two teaching methods. Since the elasticity of substitution is large, for a long range of class sizes achievement remains rather constant, because most of the time is spend on class teaching only. Once class size becomes small enough to make individual instruction possible, achievement starts to rise. The increase in achievement is substantially larger when the class becomes very small.

The other lines provide cases in which teachers divide their time according to different parameters about the effectiveness of both teaching methods. The ratio between the teachers' value and the actual relative value of both teaching methods $(\hat{\gamma}/\gamma)$ varies from 8, 1/2, 1/4, to 1/8. If teachers overestimate the value of class teaching $(\hat{\gamma}/\gamma = 8)$, pupil achievement is not seriously lowered for a wide range of class sizes. Only if the class becomes very small and individual instruction becomes an important teaching method, classes with such teachers fails to show the same increase in achievement as could be expected from an adequate division of time. To get a substantial loss in achievement under these circumstances a very large deviation between teachers' opinion and the actual value of class teaching has to be assumed.

If, on the other hand, teachers overestimate the value of individual instruction, achievement decreases rapidly for larger class sizes, while reducing size leads to lower achievement. In case ($\hat{\gamma}/\gamma = 1/2$) the decrease remains modest, but if this ratio becomes 1/4 or 1/8 the loss in achievement due to reduced classes becomes substantial. Only if classes become very small, achievement increases more than under optimal circumstances. In the limit case of a one pupil class, every class has the same achievement again.

It is interesting to note that the loss in achievement is larger when ($\hat{\gamma}/\gamma = 1/4$) then when ($\hat{\gamma}/\gamma = 1/8$). The reason for this is that due to inadequate teaching practices, achievement is already so low that further losses are limited. In relative terms, however, the case with the ratio 1/8 faces the largest decreases in achievement.

It can be concluded from this model that a positive relationship between class size and achievement might occur when class teaching and individual instruction are substitutes and teachers overestimate the value of individual instruction. The model therefore predicts that a negative impact of size reduction on achievement for some teachers can only occur when $\sigma > 1$. In the empirical analyses this is what will be tested. Furthermore, it will be investigated whether teachers' opinions about effective teaching explain variation in the relative value they attribute to class teaching and individual instruction, to explain that some school and teachers gain while others lose from class size reduction.

3 Data

To investigate the value of σ and to look at the variation in $\hat{\gamma}$, the relationship between class size and the time spend on each of the teaching methods has to be estimated as expressed in equation (11). Taking logs this equation reads

$$\ln(\hat{\theta}) = \ln\left(\frac{a}{1-an}\right) = -\sigma\ln(n) - \sigma\ln\hat{\gamma}. \tag{19}$$

Writing $\hat{\gamma}$ as a function of teacher or class characteristics (X),

$$\ln\hat{\gamma} = X\beta, \tag{20}$$

leads to

$$\ln(\hat{\theta}) = \ln\left(\frac{a}{1-an}\right) = -\sigma\ln(n) - \sigma X\beta. \tag{21}$$

To estimate this equation, data is needed about (1) class size, (2) the time allocation of teachers, and (3) information about teachers' opinions about the best way to teach. I use both data from the Class Size Reduction in California and TIMMS 1999 to obtain estimates of (21) for both primary and secondary education.

CSR data are collected by the CSR Research Consortium, monitoring the class size reduction in California, initiated in 1996. To investigate the effects of this class size reduction programm, the consortium carried out several surveys. Here I use a survey among teachers in 1998 and 2000. Detailed information about the surveys is provided by Bohrnstedt and Stecher (1999, 2000, 2002). In contrast with the STAR project in Tennessee, the CSR has not been carried out under experimental circumstances. In the course of time, schools switch from larger classes of roughly 30 to a smaller classes of roughly 20 pupils per teachers in the lower grades of the primary school.

Due to this change in policy the data contains a large variety in class sizes, with differences in size mainly caused by this policy change, rather than other exogenous factors. Some teachers have been in the survey twice, but attrition is too high to use the panel. Therefore, the estimations are based on the pooled surveys. In CSR, time use questions have been asked separately for language arts and mathematics. Both topics can be investigated separately, therefore. According to Stecher et al. (2002) no strong effect of class size reduction on achievement in mathematics and language arts are found.

The Repeated Third International Mathematics and Science Study at the Eighth Grade (TIMMS) is an initiative of the International Association for the Evaluation of Educational Achievement (IEA). In 1999 students in many countries have been tested for their abilities in mathematics and science. Together with this assessment, amongst others, teachers have been surveyed. In this paper the survey among mathematics teachers in the USA is used. Gonzalez and Miles (2001) provides detailed information about the survey. Based on TIMMS-data, Pong and Aaron (2001) claim that the USA is the only country with a positive effect of class size reduction on achievement, while Wößmann and West (2002) find – while using the same data – that in the USA there is no relationship between class size and achievement.

Both the CSR and the TIMMS survey measure class size by asking the teacher to provide the number of boys and girls in their class. Since class size might vary during the year and since teachers might make mistakes in answering, some measurement error is to be expected. Especially in TIMMS there are some very unlikely answers of teachers who claim to have classes of more than 200 pupils. A plausible reason for this is that these teachers misunderstood the question and aggregated all their students, rather than answering about the size of the class the survey focussed on. To avoid problems with class size measurement as much as possible, I excluded from the TIMMS data set all classes smaller than 15 and larger than 40. Of course, measurement error in class size will bias the estimates. Since class size is also used to construct the time spent on individual instruction per pupil, this bias will decrease the distance between σ and 1. Since $\sigma = 1$ is a crucial dividing line in the estimation, the bias has no consequences for the question whether σ is larger or smaller than 1. The size of σ might be underestimated (overestimated) when $\sigma > 1$ ($\sigma < 1$).

Both surveys also include questions about the division of time during teaching. To measure time spent on class teaching versus time spent on individual instruction, the categories distinguished in the surveys had to be classified. In CSR the teacher is asked to indicate the minutes a day (during both the teaching of language arts and mathematics) typically spent to: "the whole class"', "large groups (5 or more students)", "small groups (2-4 students)", and "individual students". Assuming that while working in groups the teachers pays attention to each group member separately, rather than teaching the group as a whole, I classified all answers except "the whole class" as individual attention. The results of the analyses turn out not to be very sensitive to this classification however.

TIMMS asks the teacher to indicate the percentage of time spent on each of the following activities in a typical month of lessons for the mathematics class: (a) "administrative tasks", (b) "homework review", (c) "lecture style presentation by the teacher", (d) "teacher-guided student practice", (e) "re-teaching and clarification of content/procedures", (f) "student independent practice", (g) "test and quizzes", and (h) "other". (a), (b) and (c) have been classified as class teaching, and the others as individual instruction. Although this classification is more arbitrary, it turns out that the results do not change substantially as long as “lecture style presentation by the teacher” is regarded as class teaching and “teacher-guided student practice” as individual attention.

In addition, both surveys also include questions about the views of the teacher about teaching. Together with variables such as grade, and sex and age of the teacher, these variables are used to explain differences in the value attributed to class teaching versus individual instruction. In CSR the following questions have been asked:

1. I mainly see my role as a facilitator. I try to provide opportunities and resources for my students to discover or construct concepts themselves.

2. I think I need to provide guidance. Although I provide opportunities for them to discover concepts, I also try to lead my students to figure things out by asking pointed questions without telling them the answers.

3. I emphasize student discussion in my classroom. We talk about concepts and problems together, exploring the meaning and/or evaluating reasoning. My role is to initiate and guide these discussions.

4. My students really won’t learn unless you go over the material in a detailed and structured way. I think it’s my job to explain, to show students how to do the work, and to give them practice in doing it.

Teachers could indicate whether they disagreed or agreed on a 1 to 5 scale.

In TIMMS opinions are asked about statements about mathematics. For the purpose of this paper the following questions seemed to be most relevant:

1. If students are having difficulty, an effective approach is to give them more practice by themselves during the class.

2. Some students have a natural talent for mathematics and others do not.

INSERT TABLE 1

Table 1 contains some basic information about the three data sets available. To get a first impression about the way teaching is affected by class size reduction, Tables 2, 3 and 4 provide the relationship between class size and detailed answers teachers in CSR gave about their lessons. The tables provide for each item the β_1-parameter of the OLS-regression $y = \beta_0 + \beta_1 \ln(n)$. A negative parameter therefore indicates an increase when class size is reduced.

INSERT TABLE 2

Table 2 provides information about general aspects of the way a teacher spends his time. Consistent with Lazear (2001), teachers in larger classes spend substantially more time on disciplining students. Furthermore, in reduced classes more time is spent on diagnosing individual needs and providing individual feedback, suggesting that indeed a shift towards individual instruction.

INSERT TABLE 3

Table 3 provides information about differences with respect to class size in teaching language arts. Three aspects of language teaching increase significantly when class size decreases. There is more work on phonics, more work in reading book and pupils write more with invented spellings. A similar table for mathematics (Table 4) shows that in smaller math classes more attention is paid to counting out loud, working with manipulatives, playing math games and using patterns to find math relationships. Larger classes use the calculator more frequently.

INSERT TABLE 4

4 Analyses

For each of the three data sets I estimated three equations relating class size to the ratio between time spend on individual instruction and class teaching. The first equation contains no control variables, the second controls for the teacher's opinion about teaching and the third controls for both the opinions and some characteristics of the teacher and the class. The results are provided in Table 5, 6 and 7.

INSERT TABLE 5

The main result is that in all three equations for all three data sets σ is significantly larger than 1. This means that when class size is reduced teachers spend more than the time saved on extra individual instruction. As a consequence pupils will get less class teaching and as long as the class size is larger than 1, the total amount of teaching time a pupil receives – i.e. time on class teaching plus his portion of the time spent on individual instruction – will also go down. Pupils will therefore only gain in achievement when the value of individual instruction is high enough to compensate this decrease in teaching time. If a teacher overestimates the value of individual instruction the relationship between achievement and size reduction might decrease substantially, as has been shown in section 2, while perhaps even a negative effect might result.

The highest value for σ is found for arts language at primary school (CSR), as indicated in Table 5. The second column shows that teachers who express the opinion that teachers have a role as facilitator, value individual instruction significantly more. Teachers who think a teacher should explain how to do the work value class teaching significantly more. Column 3 shows that the sex of the teacher and the grade of the class do not significantly change teaching practices.

INSERT TABLE 6

In teaching mathematics at the lower grades of primary school (Table 6), only those teachers who see their role as facilitator spend significantly more time on individual instruction than other teachers. There is no significant difference between teacher who think a teacher should explain and others. The third column shows that all dummies for the grades are significant lower than 0. This means that at kindergarten teachers relatively use more class teaching for mathematics than in K1-K4. Between the four grades there are no significant differences. Also no significant difference between males and females is found.

INSERT TABLE 7

In the TIMMS sample of math classes at secondary school (Table 7) teachers who think that more practice is an effective approach to overcome difficulties in learning mathematics, value individual instruction significantly more than others. Again no significant effect of age or sex of the teacher is found.

INSERT FIGURE 2

If it is true that teachers do not allocate their time optimally, but are led by ideas about the effectiveness of teaching that are not in accordance with reality, one would expect substantial differences in teaching style between teacher which are not explained by specific circumstances. To get an impression of the variation, $\hat{\gamma}$ has been predicted for all teachers, based on the estimation results in the third column, keeping the grades in the CRS sample constant at grade 1. In this way the explained part of the variation in $\hat{\gamma}$ is found, so assuming that a large part of this variation is not explained by the regression, this provides a lower bound for the actual variation.

INSERT FIGURE 3

Figure 2 shows that both for language arts and mathematics in the CRS sample a substantial variation in the relative value of individual instruction among teachers is found. For language arts this value ranges from .23 to .35, and for mathematics from .30 to .50. Figure 3 provides the same distribution for the TIMMS sample. Here also a large variation from .27 to .50 is found suggestion that teachers are far from homogenous in their ideas about optimal teaching practices.

5 Causes of Discrepancies

One of the main features of the model presented in this paper is that teachers decision how to divide their time over de two teaching methods, does not optimize the actual production function of education, but optimizes an alternative function, with $\hat{\gamma}$ rather than γ, suggesting that teachers are not fully aware about the relative value of class teaching versus individual instruction. Betts (1995) indeed argues that schools do not allocate resources efficiently, making it plausible that also in the allocation of time inefficiencies might occur.

Several questions arise from this "lack of information argument", however. First, why do people who work with pupils on a daily basis, not learn form their experiences? Second, if it is really lack of information, would a random error not be more likely, so that at least on average teachers would have the right expectations about the effectiveness of different teaching methods? In this section I will therefore discuss some alternative explanations why teachers might "overvalue" individual instruction: teachers' preferences, hidden educational targets, extra attention for low-achievers and the external effects of asking attention.

1. First, in the model presented in Section 2 the assumption is made that teachers maximize pupil achievement. Besides a good performance of their class, teachers might however also be interested in their working conditions. If working on an individual basis is more relaxed than teaching the group as a whole, teachers might deviate from the achievement maximizing behavior if competition does not force them. Arguments in the literature that increasing the incentives for teachers have a large impact on performance (Lavy 1999b), are consistent with this interpretation.

2. Second, implicitly the assumption has been made that optimizing Y is the best thing to do. An important argument for investments in education are the returns to education in the labor market. For a successful labor market position many skills or competences might be relevant, while studies like TIMMS, and the exams taken at school, focus on only a couple of aspects like reading, writing and mathematics. If there are hidden educational targets, which are important to make the educational investment successful, and not measured by tests but known by teachers, they might be right to focus also on these aims. Such hidden targets only lead to different teaching practices, however, when for these hidden targets individual instruction is relatively more important than for well-known and measured topics like reading, writing and maths.

3. Third, the assumption has been made that all pupils receive the same amount of individual instruction. In practice, however, teachers might spend a larger portion of their time to assist pupils with learning difficulties, while pupils who do well at school get less attention. If – as sug-

gested by Lazear (2001) – increasing the achievement of low-achievers is valued more than improving the performance of better students, than more weight should be given to individual instruction compared to a situation where average achievement counts. This interpretation is consistent with the findings that low-achievers benefit more from reducing class size (Hoxby 2000b, Lavy 1999b, Krueger and Whitmore 2001). The consequence would be that reducing class size decreases the achievement of better pupils, to benefit the pupils with learning difficulties.

4. Fourth, it might be difficult for teachers to avoid individual instruction. As had been put forward by Lazear (2001), class teaching has public good characteristics. If it is important for didactic reasons that students are able to ask certain questions during the lessons, a teacher has to allow interruptions, but can only afterwards judge whether the interruption was worthwhile. This provides the students with the possibility to stop the class teaching and start individual instruction without bearing the costs of time loss for others. Therefore, class rules are needed to keep class teaching going. Maintaining order in the class will cost time and energy, so in an optimum teachers will always allow more disruptions than would be optimal from a production function point of view.

6 Conclusion

This paper started with the observation that class size reductions often turn out to have less effect than might be expected *a priori* , while in many cases even a negative impact of reduced class size on achievement is found. A possible explanation for this disappointing effect is that teachers do not adjust their teaching practices adequately to new circumstances. This explanation is supported by the empirical observation that teachers increase time for individual instruction only with a small fraction. In this paper a model based on an educational production function is introduced and estimated to investigate whether suboptimal teaching practices can explain the small or negative effects of class size reductions.

The main finding is that incorrect believes of teachers about the relative value of class teaching versus individual instruction explain small or negative effects of reduced class size. Remarkable is that large deviations and possible negative effects can only occur when class teaching and individual instruction are substitutes and teachers overvalue individual instruction. Based on three data sets concerning language arts and maths at primary school and mathematics at the secondary school, consistently the required elasticity of substitution larger than 1 is found. This means that the time used for class teaching decreases while the time for individual instruction increases. Since each pupil only receives a fraction of the individual instruction provided by the teacher, the total teaching time per pupil will decrease. As a consequence, achievements will also go down, if the value of the small amount of extra individual instruction per pupil does not compensate the loss in class teaching.

An important implication is that improved efficiency of teaching, promoted e.g. by teacher training, increased incentives for teachers or ways to avoid unwanted disruption of the class teaching process by pupils, is not only an alternative way to improve investments in education, but might also be a requirement to make class size reduction (more) effective.

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Table 1

Main characteristics of the data

	CSR arts	CSR math	TIMMS
number of classes			
full set	1581	1469	320
restricted set[a]	814		
% female teachers	87.5	87.0	61.7
average class size	21.6	21.7	22.5
class size (%):			
-14 [b]	.9	.8	-
15-19	33.4	32.9	13.8
20-24	44.2	44.4	32.2
25-29	7.9	8.3	24.4
30-34	12.5	12.6	13.8
35-39	.4	.3	2.8
41+	.6	.8	-
class teaching (%)	40.8	48.8	51.2
ind. instruction (%)	59.2	51.2	48.8

[a]The CRS survey contains questions about teachers opinions about effective teaching only in the 1998 survey.

[b]Class sizes in TIMMS smaller than 15 and larger than 40 have been excluded.

Table 2

Effects of Class Size on time spend by teacher

	$\ln(n)$	
disciplining students	.468	(.136)*
monitoring progress	-.141	(.154)
doing paperwork	.183	(.225)
diagnosing individual needs	-.457	(.173)*
reviewing homework	.128	(.163)
providing individual feedback	-.283	(.161)*
addressing personal concerns	-.244	(.154)
discussing student-initiated topics	-.183	(.136)

Table 3

Effects of Class Size on way language arts are taught

	ln(n)	
practice writing alphabet	-.063	(.250)
dictate stories to teacher	-.173	(.192)
work on phonics	-.472	(.188)*
listen and follow printed stories	-.282	(.201)
work in reading book	-.732	(.200)*
write with invented spellings	-.389	(.174)*
read self-chosen books	-.064	(.085)
write narrative pieces	-.191	(.168)
discuss reading	-.199	(.145)
use computer	-.076	(.185)
read aloud to partner	-.236	(.189)
take test or quiz	.063	(.124)

Table 4

Effects of Class Size on way mathematics is taught

	ln(n)	
count out loud	-.730	(.219)*
work with manipulatives	-.504	(.169)*
play math games	-.394	(.153)*
use calculator	.320	(.145)*
use computer	-.097	(.174)
work with measuring instruments	-.094	(.135)
explain math solution	-.277	(.173)
do math worksheets	.192	(.172)
complete math problems on board	.189	(.199)
practice computational skills	-.103	(.134)
present tables, graphs, pictures	-.279	(.153)
use math with other subjects	-.167	(.160)
use patterns to find math relationships	-.500	(.164)*
take math test, quiz	.138	(.150)

Table 5

Teachers' implicit production function for teaching language arts at primary school

	1		2		3	
σ	1.495	(.100)*	1.395	(.120)*	1.320	(.126)*
$\ln(\hat{\gamma})$:						
constant	-1.287	(.119)*	-1.264	(.162)*	-1.067	(.224)*
role as facilitator			-.047	(.021)*	-.048	(.022)*
provide guidance			-.004	(.028)	-.006	(.029)
emphasize discussion			.021	(.025)	.022	(.027)
explain			.040	(.017)*	.043	(.018)*
female					-.016	(.063)
1st grade					-.128	(.101)
2nd grade					-.134	(.079)
3rd grade					-.038	(.092)
4th grade					-.069	(.112)
R^2	.124		.148		.155	

Table 6

Teachers' implicit production function for teaching mathematics at primary school

	1		2		3	
σ	1.288	(.109)*	1.349	(.135)*	1.345	(.140)*
$\hat{\gamma}$:						
constant	-.716	(.199)*	-.836	(.229)*	-.471	(.299)
role as facilitator			-.076	(.025)*	-.071	(.024)*
provide guidance			.032	(.032)	.028	(.032)
emphasize discussion			-.043	(.030)	-.041	(.030)
explain			.005	(.019)	.005	(.019)
female					-.049	(.065)
1st grade					-.343	(.115)*
2nd grade					-.226	(.089)*
3th grade					-.291	(.104)*
4th grade					-.286	(.119)*
R^2	.086		.132		.150	

Table 7

Teachers' implicit production function for teaching mathematics at secondary school

	1		2		3	
σ	1.207	(.165)*	1.263	(.168)*	1.209	(.167)*
$\hat{\gamma}$:						
constant	-.947	(.308)*	-1.02	(.326)*	-.661	(.463)
practice themselves			-.121	(.041)*	-.126	(.044)*
natural talents			.073	(.044)	.083	(.046)
age					-.097	(.141)
age square					-.010	(.019)
female					-.108	(.062)
R^2	.169		.202		.209	

Figure 1

Numerical example of the model illustrating the possible relationships between class size and achievement (ρ=0.8 , γ_a=1 , and γ_c=0.8 , with $\hat{\gamma}/\gamma$ varies from 8 to 1/8)

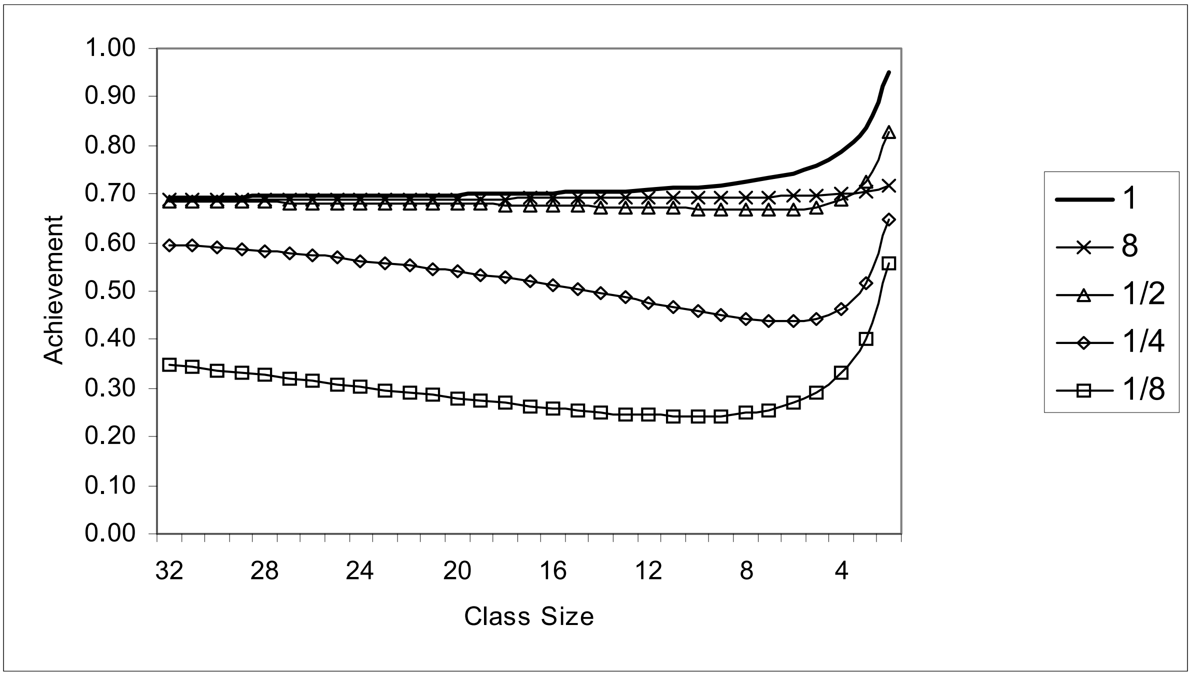

Figure 2
Simulated distribution of $\hat{\gamma}$ for language arts and mathematics at primary school (CSR sample)

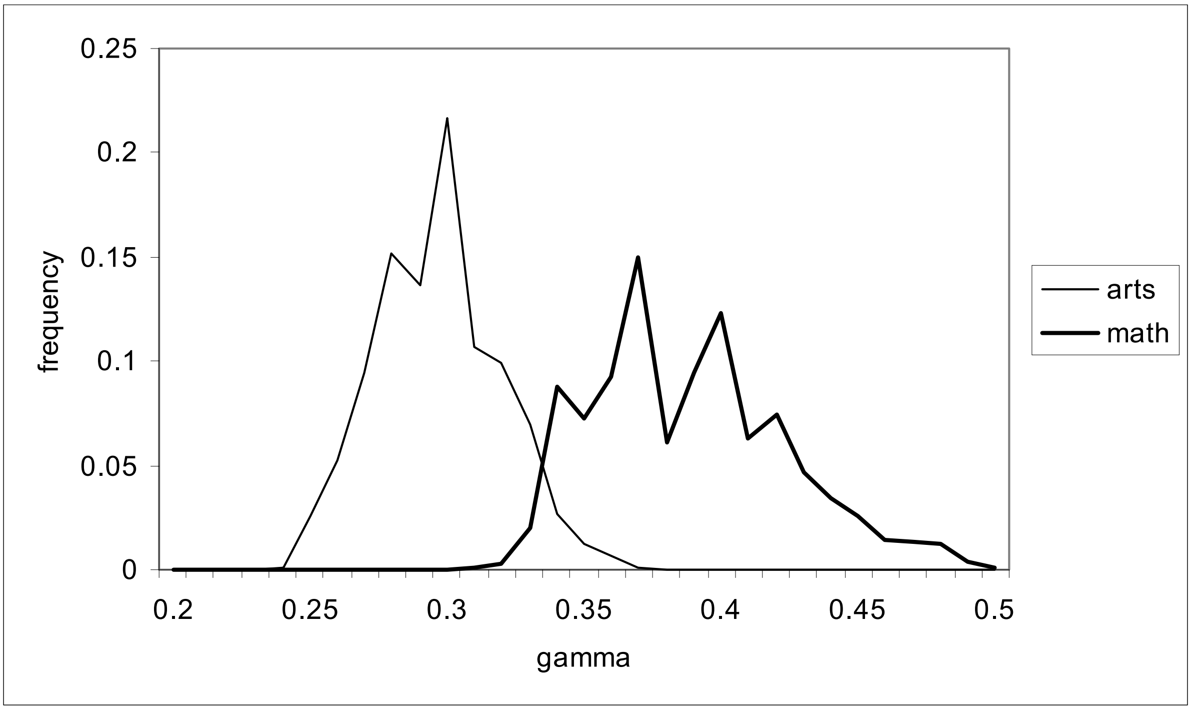

Figure 3

Simulated distribution of $\hat{\gamma}$ for mathematics at secondary school (TIMMS sample)